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Thermohydraulic performance of a periodic trapezoidal channel with a triangular cross-section

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Abstract

Simulations are performed to study the heat transfer behaviour of an equilateral triangular section duct following a tortuous path for fully-developed laminar flows with Reynolds numbers below 200. The enhancement of heat transfer and the increase in pressure drop are compared with those for ducts of circular, semi-circular and square section following the same serpentine path. For this flow regime, the triangular duct is shown to be the optimum choice (best heat transfer augmentation compared with increased pressure drop) amongst those studied. The effects of changing the path shape, the apex angle for an isosceles triangular cross-section and rounding of a corner of the equilateral triangular duct are also considered.

Keywords: Heat transfer enhancement; Triangular duct; Micro heat exchanger; Laminar flow

1. Introduction

Forced convective heat transfer in triangular passages is of considerable interest in a wide variety of applications and especially in the design of heat exchangers. Traditionally, it is the analysis of plate and fin heat exchangers that has provided the driver for the study of this geometry but designers of solar collectors and compact heat exchangers also require pressure drop and heat transfer data for triangular section channels. As far back as the late 50s and early 60s, Eckert et al. [1], Sparrow [2], Sparrow and Haji-sheikh [3] and Schmidt and Newell [4] used approximate solution methods to study the pressure drop and convective heat transfer in fully-developed laminar flow in ducts with cross-sections having an equilateral or isosceles triangular section.

Shah [5] and Shah and London [6] studied the heat transfer characteristics of laminar flow in a wide variety of channel shapes, including for equilateral triangular, equilateral triangular with rounded corners, isosceles triangular, right triangular and arbitrary triangular cross-section ducts, for an extensive range of thermal boundary conditions. Since then the improvement in computational capabilities has led to the study of laminar, fully-developed flow in triangular plate-fin ducts including heat conduction through the fins [7], flow in triangular section ducts used in solar collectors that have different thermal boundary conditions on some faces [8], and flow in cross-corrugated triangular ducts [9]. Chen et al. [10] studied numerically the flow and heat transfer characteristics of smooth triangular ducts with different apex angles for fully-developed laminar flow conditions. They found that the apex angle of 60° provided the highest steady-state forced convection heat transfer coefficient. Recently, Zhang [11] has reported Nusselt numbers for laminar hydrodynamically fully-developed and thermally-developing flow for a uniform wall temperature condition in isosceles triangular ducts with apex angles ranging from 30° to 120°. Experimental studies of laminar and turbulent flow in triangular ducts [12], forced convective heat transfer [13–15] and forced convection in triangular ducts containing fins [16] have also been performed to complement the simulation work.

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Nomenclature

A	amplitude of the trapezoidal path (m)
B	length of the short side of the trapezoid (m)
d	channel diameter or length of side in equilateral triangle case (m)
d_h	hydraulic diameter (m)
e	dimensionless enhancement/penalty factor
f	friction factor
h	heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
L	trapezoidal unit half-wavelength (m)
Nu	Nusselt number ($=hd_h/\alpha$)
R_c	radius of curvature (m)
Re	Reynolds number ($=Vd_h/\upsilon$)
V	Average velocity in duct (ms^{-1})

Greek symbols

α	thermal diffussivity ($m^2 s^{-1}$)
ΔP	pressure drop (Pa)
η	efficiency ($=e_{Nu}/e_f$)
υ	kinematic viscosity ($m^2\ s^{-1}$)
ρ	fluid density ($kg\ m^{-3}$)

Subscripts

f	pressure drop penalty factor
Nu	heat transfer enhancement factor
straight	quantity in straight channel
trapezoidal	quantity in trapezoidal channel

We have previously computed fully-developed laminar flow and heat transfer in serpentine, sinusoidal and trapezoidal passages having square, circular and semi-circular cross-sections [17–22]. It was observed that the formation of Dean vortices at the bends enhances the mixing of fluid in the channel, which in turn enhances heat transfer. The pressure drop penalty incurred in using such passages is low compared with the heat transfer augmentation, making them beneficial in such applications. This paper investigates the performance of equilateral triangular cross-section ducts following serpentine and trapezoidal paths for various Reynolds numbers and for the constant wall heat flux (H2) boundary condition. We also study the effect of rounding one corner and changing the apex angle for isosceles triangular section ducts. This cross-section shape is important as it arises in many methods used to generate micro-channels paths for compact heat exchangers. Table 1 shows values of the friction factor and Nusselt number for straight channels of different cross-sections [6] that are used to normalize the tortuous path data.

2. Computational method

The trapezoidal passage shown in Fig. 1 is defined by sweeping an equilateral triangle along the chosen path. This trapezoidal path is similar to the one used previously to analyse other cross-sections [19]. The case $B/L = 1$ results in a serpentine path which has been used to study the effect of Reynolds number on the heat transfer enhancement. $A/L = 0$ corresponds to the limiting case of a straight duct, and the value $R_c/d = 0.5$ corresponds to a sharp corner (zero radius of curvature) at the inside of any bend.

Table 1
Reference values of the friction factor and Nusselt number used to normalize the current results

Cross-section	fRe	Nu_{H2}
Square	14.227	3.091
Circular	16	4.363
Semicircular	15.767	2.923
Triangular (equilateral)	13.333	1.892
Triangular (one corner rounded)	14.057	2.196

Data are taken from [6].

Steady-state simulations were carried out using ANSYS CFX 11 and the methodology described in [17,18] for a fluid with constant properties and a Prandtl number of 6.13 (water). The identical solution methodology and high order numerical approach to that reported early is used to obtain fully-developed flow and heat transfer data for this cross-sectional shape. A structured hexahedral mesh was used for the discretization of the system. The cross-sectional mesh comprising typically 1875 elements was biased towards the walls and it was tested to ensure that it gave a grid-independent solution. The longitudinal mesh density was set such that the node groups were distributed relatively evenly along the flow axis. Trapezoidal paths used in previous work [19] contained approximately 120–160 element groups along the flow axis of each channel which was shown to provide grid-independent solutions. There-

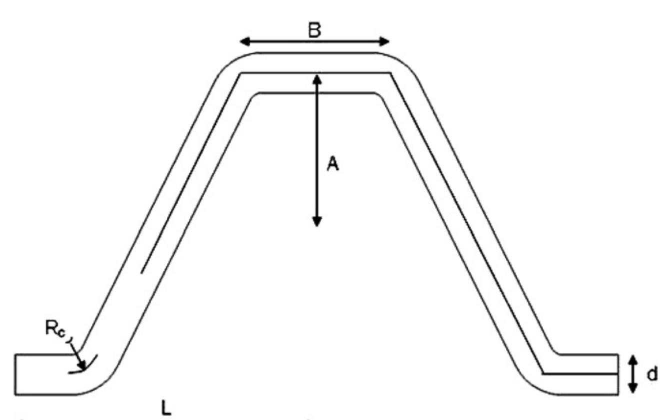

Fig. 1. The dimensions of the trapezoidal unit. L is the unit half length, d is the length of the side of the triangle, R_c is the radius of curvature, A is the half height of the trapezoid and B is the length of the top run.

fore, a similar longitudinal mesh density was used for these simulations. In order to confirm the accuracy of the method, the friction factor and Nusselt number for laminar fully-developed flow and heat transfer in straight ducts were calculated, and were found to be within 0.05% and 0.1%, respectively, of the values quoted in Table 1.

Heat transfer and pressure drop data were calculated from the converged solution as previously described. Heat transfer results are expressed in the form of heat transfer enhancement, e_{Nu}, as defined in Eq. (1), while the pressure drop penalty has been expressed in terms of e_f, as defined in Eq. (2)

$$e_{\mathrm{Nu}} = \frac{Nu_{\mathrm{channel}}}{Nu_{\mathrm{straight}}} \tag{1}$$

$$e_{\mathrm{f}} = \frac{\Delta P_{\mathrm{channel}}}{\Delta P_{\mathrm{straight}}} = \frac{f_{\mathrm{channel}}}{f_{\mathrm{straight}}} \tag{2}$$

3. Results and discussion

3.1. Ducts following a serpentine path

Fully-developed laminar flow and heat transfer simulations for the constant wall heat flux (H2) boundary condition were carried out for an equilateral triangular-shaped duct section and a serpentine path ($R_c/d = 1$, $L/d = 4.5$, $A/L = 0.5$ and $B/L = 1$) for different Reynolds numbers. The values of e_{Nu} and e_f were compared with those obtained for other cross-sections following the same serpentine path. Fig. 2a and b show the comparison of heat transfer enhancement and pressure drop penalty for the triangular cross-section with different shaped cross-sections [20]. The heat transfer enhancement for triangular cross-section ducts is comparable with that achieved for the square cross-section, which has the best heat transfer performance amongst all the other cross-sections. The pressure drop penalty for equilateral triangular cross-section ducts is comparable with the pressure drop penalty for semi-circular cross-section ducts which has the minimum pressure drop penalty amongst all the cross-sections studied. Thus, the triangular cross-section offers the advantage of better heat transfer enhancement coupled with low pressure drop penalty as compared with other cross-sections.

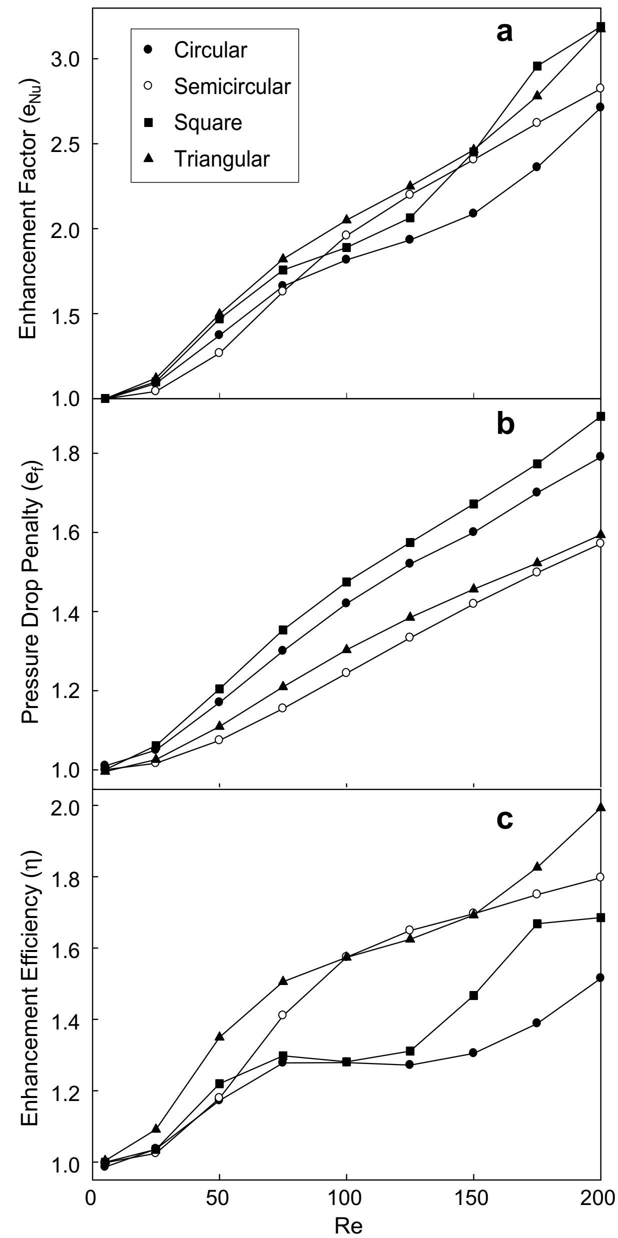

Fig. 2. Heat transfer enhancement (a), pressure drop penalty (b) and enhancement efficiency (c) as a function of Reynolds number for different cross-sections ($R_c/d = 1.0$, $L/d = 4.5$, $A/L = 0.5$, $B/L = 1.0$).

Fig. 2c compares the efficiency (the ratio of heat transfer enhancement to pressure drop penalty), η, of ducts of different cross-sections over a range of Reynolds number. The triangular duct has an efficiency greater than one even at low Reynolds number which is not observed in the ducts of other cross-sections. At high Reynolds numbers, the triangular duct gives the maximum efficiency. Thus, the triangular duct offers the best efficiency when considered over the entire Reynolds number range.

For this geometry, the flow field near the bends contains two vortices, one on top of the other. Dean vortices, very similar to those observed in semi-circular ducts, bring the fluid from the walls towards the centre, increasing the heat transfer rate. A transition from two to four vortices as the Reynolds number is increased over the range covered here is not observed, unlike the situation in the circular and square section channels. The volume averaged helicity increases monotonically with Reynolds number as observed in semi-circular ducts [17,20].

Fully-developed flow and heat transfer analyses were also carried out for a serpentine duct ($B/L = 1$) whose cross-section is that of an equilateral triangle with one rounded corner (the centre of the arc was located at a distance $d/3$ from the apex at the base of the triangle, where d is the length of the side of the triangle). The straight triangular duct with one rounded corner has a Nusselt number

Table 2
Performance parameters for ducts of different triangular cross-sections following a serpentine path ($R_c/d = 1.0$, $L/d = 4.5$, $A/L = 0.5$) at $Re = 200$

Cross-section	Nu	e_{Nu}	e_f	η
Triangular (equilateral)	5.935	3.137	1.592	1.970
Triangular (one corner rounded)	6.093	2.775	1.513	1.834

higher by ~16% than the same duct with no rounded corner. Table 2 compares the performance of triangular serpentine ducts with sharp and rounded corner for a Reynolds number of 200. As for straight ducts, the Nusselt number is higher for a duct with a rounded corner because of the elimination of the dead zone near the apex of the triangle. However, the difference between the results for sharp and the rounded corner profile is somewhat less for the serpentine pathway than for the straight passages, such that the enhancement due to tortuosity is less when one corner is rounded.

3.2. Effect of path shape

We have also analysed the equilateral triangular duct following trapezoidal paths with different values of B/L and compared the results with those obtained earlier for semi-circular ducts [19]. Fig. 3 shows the heat transfer enhancement and pressure drop penalty as a function of B/L. Changing B/L has little influence on the heat transfer and pressure drop penalty in triangular ducts, as was also found for semi-circular ducts.

3.3. Effect of apex angle

The effect of changing from an equilateral triangular cross-section to an isosceles triangle was also investigated. In these simulations the length of the flat surface at the top of the duct was kept constant, the angle of the apex was varied between 30° and 120° for a Reynolds number of 200 and for a trapezoidal path having $R_c/d = 0.8$, $L/d = 4.5$, $A/L = 0.5$ and $B/L = 0.5$, as depicted in Fig. 1. Fig. 4a shows the Nusselt numbers for the fully-developed flow in a straight section (taken from Shah and London [6]) and for the tortuous path. The effect of the trapezoidal pathway is to decrease the optimal angle to below 50°. Fig. 4b shows the heat transfer enhancement and pressure drop penalties. The heat transfer enhancement is least at 60°. At the 30° and 120° apex angles, three Dean vortices were observed across the cross-section, as compared with two at 60° which accounts for the higher heat transfer enhancement at these angles. Although the heat transfer enhancement is weakest for the equilateral triangle, the Nusselt number and so heat transfer performance is near the maximum, which occurs in the apex angle range 40–60°.

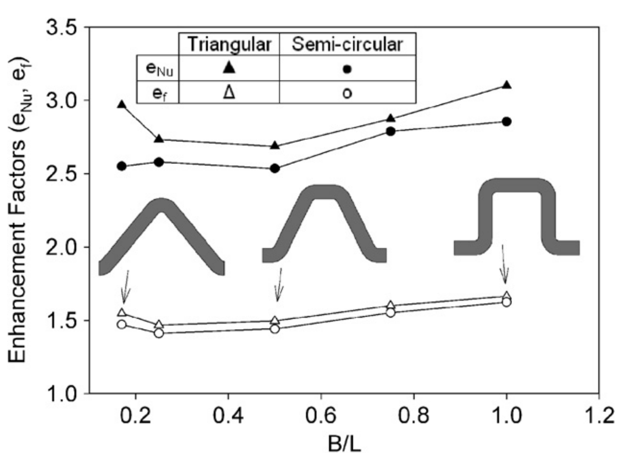

Fig. 3. Heat transfer enhancement and relative pressure drop penalty as a function of B/L for a trapezoidal path ($R_c/d = 0.8$, $L/d = 4.5$, $A/L = 0.5$, $Re = 200$).

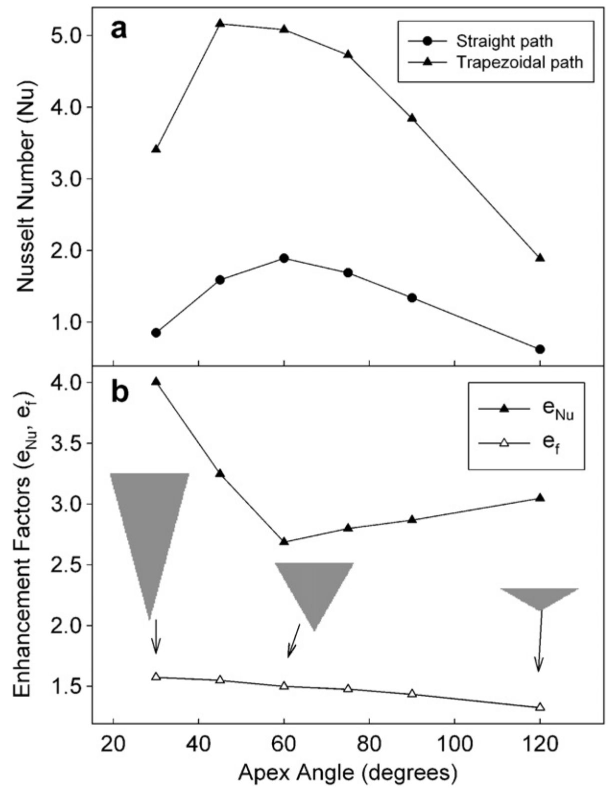

Fig. 4. Nusselt number (a) and heat transfer enhancement and relative pressure drop penalty (b) as a function of apex angle for isosceles triangular cross-section following a trapezoidal path ($R_c/d = 0.8$, $L/d = 4.5$, $A/L = 0.5$, $B/L = 0.5$, $Re = 200$).

4. Conclusion

Fully-developed laminar flow and heat transfer in equilateral triangular cross-section ducts following serpentine and trapezoidal path has been studied. For a serpentine path the triangular duct was found to have superior heat transfer performance than ducts of circular, semi-circular and square cross-sections. The equilateral triangular section duct exhibits an enhancement efficiency greater than unity even at low Reynolds number, which is not observed in the ducts of other cross-sections. This high efficiency in triangular ducts can be attributed to the complex flow pattern developed in the form of two Dean vortices. Rounding the corner of the triangle does not offer any advantage in terms of the heat transfer enhancement for triangular tortuous ducts. For fully-developed laminar flow and heat transfer equilateral triangular ducts offer the best overall heat transfer enhancement compared with those having circular, semi-circular and square cross-sections. The heat transfer performance was found to depend weakly on the path shape (varied from a zigzag to a serpentine) but to be much more sensitive to the apex angle for an isosceles triangle section duct.

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